Sonic-Boom Prediction with Output-Based Adaptation and Cart3D

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Problem Description

• Objective is ground signal

• Basic approach:
  ‣ Compute accurate pressures in the “near-field”
  ‣ Propagate to ground using atmospheric propagation code

• Fundamental difficulty
  ‣ Can be expensive due to long propagation distances
Approach

• Use adjoint-based mesh adaptation with Cart3D

• Drive adaptation with signal at off-body sensor in near/mid field

• Make every attempt to minimize expense of computation
Outline

• Method & Development history

• Basics of method
  ‣ Generic example
  ‣ Specialization for boom

• Workshop Examples
  ‣ Results & comparisons
  ‣ Cell-counts
  ‣ Timings
Cart3D: Overview

Cut-Cell Cartesian Method
- Fully-automated mesh generation from watertight geometry
- Unstructured Cartesian cells
- Insensitive to geometric complexity
- Multigrid accelerated upwind scheme

Highly Scaleable
- Domain decomposition
- “On-the-fly” mesh partitioning w/ SFC-based partitioner
- OpenMP and MPI builds
- Excellent scalability on Columbia, Pleiades and RTJones
Cart3D: Mesh Adaptation

- Basic adaptation infrastructure for Cart3D developed in 2001-’02

- Adjoint approach involves solution of flow eqs. & corresponding adjoint eqs.

- Main Benefits:
  - **Efficiency:** Focus only on discretization error which impacts performance (functional)
  - **Credibility:** Every simulation includes:
    1. Mesh refinement study to demonstrate mesh convergence
    2. Adjoint correction term to functional
    3. Bound on remaining error in discrete solution
  - **Goal is a user independent predictive tool!**
    - Remove dependence on “expert knowledge” to generate good mesh
    - Even “expert” learns from final mesh
    - Remove user bias that even expert brings to meshing
Cart3D: Adjoint Development

Boom Prediction:

Adjoint-Based Adaptation:

Adjoint Method for Cut-cell Cartesian Meshes
Method Example

- NACA 0012 airfoil
  - $M_\infty = 0.8$
  - $\alpha = 1.25^\circ$

- Functional: $C_D$

- TOL: 4 counts
Method Example

1. Compute flow solution
2. Compute adjoint solution
3. Compute adjoint correction
4. Compute cell-wise error $e_k$

Net error: $E = \sum_{k=0}^{N} e_k$
Method Example

1. Compute flow solution
2. Compute adjoint solution
3. Compute adjoint correction
4. Compute cell-wise error $e_k$

Net error: $E = \sum_{k=0}^{N} e_k$

5. Refine mesh where cell-wise error exceeds threshold
Method Example

1. Compute flow solution
2. Compute adjoint solution
3. Compute adjoint correction
4. Compute cell-wise error $e_k$
   
   Net error:  
   \[ E = \sum_{k=0}^{N} e_k \]
5. Refine mesh where cell-wise error exceeds threshold
6. If ($E < TOL$) Stop
Method Example

Functional convergence

Estimate of Remaining Error

- Functional
- Corrected Functional

$C_D$

Error Estimate

Number of Cells

$10^3$ $10^4$

$10^3$ $10^4$
Method Example

Functional convergence

Corrected functional accurately predicts next answer. Error steadily decreases.
Specialization for Sonic-Boom

• Re-examine simulation setup

• Signals propagating from body are measured along near-field sensor

• “Squared functional” used for sensor

\[ J_s = \int_0^L \left( \frac{\Delta p}{p_\infty} \right)^2 ds \]

- Introduced in AIAA-2008-0725
- Emphasizes peaks
- Vanishing derivative near \( \Delta p = 0 \)
Specialization for Sonic-Boom

- Traditional problem layout
- Cartesian-aligned edges
- Cubic (isotropic) cells
Specialization for Sonic-Boom

- To enhance signal propagation towards the sensor:
  
  Rotate mesh by Mach-angle, $\mu$

  \[
  \mu = \sin^{-1} \left( \frac{1}{M_\infty} \right) 
  \]
Specialization for Sonic-Boom

• To enhance signal propagation towards the sensor:

  Rotate mesh by Mach-angle, $\mu$

  $$\mu = \sin^{-1} \left( \frac{1}{M_\infty} \right)$$
Specialization for Sonic-Boom

• To enhance signal propagation towards the sensor:
  Rotate mesh by Mach-angle, $\mu$

  $$\mu = \sin^{-1}\left(\frac{1}{M_\infty}\right)$$

• Stretch cells to increase per-cell propagation distance
Specialization for Sonic-Boom

• To enhance signal propagation towards the sensor:
  
  Rotate mesh by Mach-angle, \( \mu \)

  \[ \mu = \sin^{-1} \left( \frac{1}{M_{\infty}} \right) \]

• Stretch cells to increase per-cell propagation distance

*Rotation & stretching give substantial savings, see Full investigations in AIAA 2008-0725 & 2008-6593*
Results Overview

- Axisymmetric bodies
  - 6.48° Cone-cylinder
  - Parabolic
  - Quartic

- 69° Swept Delta-wing-body

- Ames Low Boom Wing Tail with Nacelles
Results Overview

• Axisymmetric bodies
  ‣ 6.48° Cone-cylinder
  ‣ Parabolic
  ‣ Quartic

• 69° Swept Delta-wing-body

• Ames Low Boom Wing Tail with Nacelles

All cases run “hands-off” starting from: surface triangulation, mesh bounding box & error tolerance
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$
  - $\alpha = 0.0°$
  - Sensor offset, $h/L = 10.0$
- Initial mesh ~ 6300 cells
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$
  - $\alpha = 0.0^\circ$
  - Sensor offset, $h/L = 10.0$
- Initial mesh $\sim 6300$ cells
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$
  - $\alpha = 0.0°$

$L = 8.6$

Isobars

6300 cells
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$
  - $\alpha = 0.0°$

Isobars

Signal at sensor

6300 cells

Distance Along Sensor (x/L)

$\Delta p/p_\infty$
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$
  - $\alpha = 0.0°$

- Initial mesh ~ 6300 cells

- Isobars

- 3 adaptations

- Distance Along Sensor ($x/L$)

- $\Delta p/p_\infty$
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_{\infty} = 1.68$
  - $\alpha = 0.0^\circ$

Isobars

2.4 M cells

Distance Along Sensor ($x/L$)

Delta $p/p_{\infty}$

9 adaptations
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$
  - $\alpha = 0.0°$

Isobars

3.29M cells
6.48° Cone-Cylinder

- NASA TM X-2219
  - \( M_\infty = 1.68 \)
  - \( \alpha = 0.0° \)

\[ L = 8.6 \]

Isobars

13 adaptations

Cart3D: 3.29 M cells

Experiment, \( h/L = 10. \)
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68$, $\alpha = 0.0^\circ$, $h/L = 10.0$

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**Functional Convergence**

\[ J_s = \int_0^L \left( \frac{\Delta p}{p_\infty} \right)^2 ds \]

- Functional
- Corrected Functional

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**Error Convergence**

- Mesh Adaptation
- Error Tolerance
  - Satisfy tolerance

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**Normalized Error Estimate**

<table>
<thead>
<tr>
<th>Number of Cells</th>
<th>Functional Convergence Error Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10^4</td>
</tr>
<tr>
<td>1</td>
<td>10^5</td>
</tr>
<tr>
<td>2</td>
<td>10^6</td>
</tr>
<tr>
<td>3</td>
<td>10^7</td>
</tr>
<tr>
<td>4</td>
<td>10^8</td>
</tr>
<tr>
<td>5</td>
<td>10^9</td>
</tr>
<tr>
<td>6</td>
<td>10^10</td>
</tr>
</tbody>
</table>

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30
6.48° Cone-Cylinder

- NASA TM X-2219
  - $M_\infty = 1.68, \alpha = 0.0^\circ, h/L = 10.0$

- Simulation performed on desktop workstation
  - Dual quad-core (8 cores)
  - Intel Xeon, 3.2Ghz
  - 16 Gb memory

- Total simulation time 41 mins. (all adaptations & mesh gen)

![Pie chart showing simulation time distribution]

- Adjoint Solves: 39%
- Flow Solves: 54%
- Mesh Adaptation: 7%

Total = 41 mins.
Parabolic: \( r = f(x^{1/2}) \)

- NASA TN D-3106
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)
  - Sensor offset, \( h/L = 10.0 \)
- Initial mesh \(~3200\) cells

\[ L = 2.0 \]
Parabolic: \( r = f(x^{1/2}) \)

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  - \( M_\infty = 1.41 \)
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\( L = 2.0 \)

\( h = 10L \)
Parabolic: \( r = f(x^{1/2}) \)

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\[ L = 2.0 \]

\[ h = 10L \]

Isobars

3.58 M cells
Parabolic: \( r = f(x^{1/2}) \)

- NASA TN D-3106
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)

\[ L = 2.0 \]
Parabolic: \( r = f(x^{1/2}) \)

- NASA TN D-3106
  - \( M_\infty = 1.41, \alpha = 0.0^\circ \)
  - \( h/L = 10.0 \)

- Simulation performed on desktop workstation
  - Dual quad-core (8 cores)
  - Intel Xeon, 3.2Ghz
  - 16 Gb memory

- Total simulation time 75 mins. (all adaptations & mesh gen)
Quartic: \( r = f(x^{1/4}) \)

- NASA TN D-3106
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)
  - Sensor offset, \( h/L = 10.0 \)
- Initial mesh \( \sim 3200 \) cells

\( L = 2.0 \)
Quartic: \( r = f(x^{1/4}) \)

- NASA TN D-3106
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)
  - Sensor offset, \( h/L = 10.0 \)
- Initial mesh \( \sim 3200 \) cells
Quartic: \( r = f(x^{1/4}) \)

- **NASA TN D-3106**
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)
  - Sensor offset, \( h/L = 10.0 \)
- Initial mesh \( \sim 3200 \) cells
Quartic: \( r = f(x^{1/4}) \)

- NASA TN D-3106
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)

\[ L = 2.0 \]

\[ h = 10L \]

Isobars

3200 cells
Quartic: \[ r = f(x^{1/4}) \]

- NASA TN D-3106
  - \( M_\infty = 1.41 \)
  - \( \alpha = 0.0^\circ \)

Isobars

3.98 M cells

\[ h = 10L \]

\[ L = 2.0 \]
Quartic: $r = f\left(x^{1/4}\right)$

- NASA TN D-3106
  - $M_\infty = 1.41$
  - $\alpha = 0.0^\circ$

\[ L = 2.0 \]

Isobars

Cart3D: 3.98 M cells
Experiment, $h/L = 10.$

\[ \frac{\Delta p}{p_\infty} \]
Quartic: \[ r = f(x^{1/4}) \]

- **NASA TN D-3106**
  - \( M_\infty = 1.41, \alpha = 0.0^\circ \)
  - \( h/L = 10.0 \)

- Simulation performed on desktop workstation
  - Dual quad-core (8 cores)
  - Intel Xeon, 3.2Ghz
  - 16 Gb memory

- Total simulation time 83 mins. (all adaptations & mesh gen)

**Total = 83 mins.**
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_\infty = 1.68$
  - $\alpha = 4.74°$
  - Sensor offset, $h/L = 3.6$ &
    \{0.2, 0.4, 0.8, 1.2, 2.0, 2.8\}

- Initial mesh ~ 22 k cells
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_\infty = 1.68$
  - $\alpha = 4.74^\circ$
  - Sensor offset, $h/L = 3.6$ &
    - {0.2, 0.4, 0.8, 1.2, 2.0, 2.8}

- Initial mesh ~ 22 k cells

Stepped sting-body juncture
69° Swept Delta Wing-Body

- NASA TN D-7160
  - \( M_\infty = 1.68 \)
  - \( \alpha = 4.74^\circ \)
  - Sensor offset, \( h/L = 3.6 \) & \( \{0.2, 0.4, 0.8, 1.2, 2.0, 2.8\} \)

- Initial mesh ~ 22 k cells
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_\infty = 1.68$
  - $\alpha = 4.74^\circ$

Isobars

22 k cells
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_{\infty} = 1.68$
  - $\alpha = 4.74^\circ$

Isobars

2.26 M cells
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_\infty = 1.68$
  - $\alpha = 4.74°$

Isobars

- Initial mesh ~ 22k cells
- $L = 17.52$
- $2.26M$ cells

Sensor offset, $h/L = 3.6$ & {0.2, 0.4, 0.8, 1.2, 2.0, 2.8}
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_\infty = 1.68$
  - $\alpha = 4.74^\circ$

Isobars

Cart3D: 2.26 M cells
Experiment, $h/L = 3.6$
69° Swept Delta Wing-Body

2.26M cells
69° Swept Delta Wing-Body

- NASA TN D-7160
  - $M_\infty = 1.68$
  - $\alpha = 4.74°$
  - $h/L = \{.2, .4, .8, 1.2, 2.0, 2.8, 3.6\}$

- Simulation performed on desktop workstation
  - Dual quad-core (8 cores)
  - Intel Xeon, 3.2Ghz
  - 16 Gb memory

- Total simulation time 53 mins. (all adaptations & mesh gen)

Total = 53 mins.
Ames Low-Boom Wing Tail

- NASA CP-1999-209699
  - $M_\infty = 2.0$
  - $\alpha = 2.0^\circ$
  - Sensor offset, $h/L = 1.167$
- Initial mesh $\sim 111$ k cells
Ames Low-Boom Wing Tail

- NASA CP-1999-209699
  - \( M_\infty = 2.0 \)
  - \( \alpha = 2.0^\circ \)
  - Sensor offset, \( h/L = 1.167 \)
- Initial mesh ~ 111 k cells

Stepped sting-body juncture

\( L = 12.0 \)
Ames Low-Boom Wing Tail

- NASA CP-1999-209699
  - $M_\infty = 2.0$
  - $\alpha = 2.0^\circ$
  - Sensor offset, $h/L = 1.167$
- Initial mesh ~ 111 k cells
Ames Low-Boom Wing Tail

- NASA CP-1999-209699
  - $M_\infty = 2.0$
  - $\alpha = 2.0^\circ$

![Ames Low-Boom Wing Tail Diagram with Isobars and 7.20 M cells]
Ames Low-Boom Wing Tail

- NASA CP-1999-209699
  - $M_{\infty} = 2.0$
  - $\alpha = 2.0^\circ$

Isobars
Ames Low-Boom Wing Tail

- NASA CP-1999-209699
  - $M_\infty = 2.0$
  - $\alpha = 2.0^\circ$
  - Sensor offset, $h/L = 1.167$

- Simulation performed on desktop workstation
  - Dual quad-core (8 cores)
  - Intel Xeon, 3.2Ghz
  - 16 Gb memory

- Total simulation time 90 mins. (all adaptations & mesh gen)

- Total = 90 mins.
## Mesh Sizes and Computing Resources

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Mach</th>
<th>AoA</th>
<th>h/L</th>
<th>Num. Control Volumes (on final mesh)</th>
<th>Net wallclock time (mins)</th>
<th>Net CPU time (mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.48° Cone-Cylinder NASA TM X-2219</td>
<td>1.68</td>
<td>0°</td>
<td>10</td>
<td>$3.29 \times 10^6$</td>
<td>41 mins</td>
<td>328 mins</td>
</tr>
<tr>
<td>Parabolic Body of Revolution NASA TN D-3106</td>
<td>1.41</td>
<td>0°</td>
<td>10</td>
<td>$3.58 \times 10^6$</td>
<td>75 mins</td>
<td>600 mins</td>
</tr>
<tr>
<td>Quartic Body of Revolution NASA TN D-3106</td>
<td>1.41</td>
<td>0°</td>
<td>10</td>
<td>$3.98 \times 10^6$</td>
<td>83 mins</td>
<td>664 mins</td>
</tr>
<tr>
<td>69° Swept Delta Wing-Body NASA TN D-7160</td>
<td>1.68</td>
<td>4.74°</td>
<td>3.6</td>
<td>$2.26 \times 10^6$</td>
<td>53 mins</td>
<td>424 mins</td>
</tr>
<tr>
<td>Ames Low Boom Wing Tail with Nacelles NASA CP-1999-209699</td>
<td>2.0</td>
<td>2.0°</td>
<td>1.167</td>
<td>$7.20 \times 10^6$</td>
<td>90 mins</td>
<td>720 mins</td>
</tr>
</tbody>
</table>

1 Delta wing body results at $h/L = \{0.2, 0.4, 0.8, 1.2, 2.0, 2.8, 3.6\}$, experimental data at $h/L = 3.6$ only

2 All simulations on desktop workstation with dual quad-core (8 cores) Xeon processors, 16Gb memory
Summary and Future work

• Basic approach seems sound
  ‣ Very good agreement with experiment for variety of geometry and conditions.
  ‣ Robust and automatic, all cases same CFL, same limiter.
  ‣ Reasonable turnaround time on commodity hardware.
  ‣ 1-2 hrs on 8 cores for all workshop problems
  ‣ Very economical! Workshop examples required from 2.3-7.2 M cells
  ‣ Longer propagation distances and complex geometry easily within reach

• Best objective function?
  ‣ Won’t know until we start propagating signals to ground. Even then…
  ‣ What are most important properties of near field signal?
  ‣ What are acceptable boom profiles? dBA? Hardest on buildings?

• No issues outstanding before refocusing on propagation and shape design
Questions?